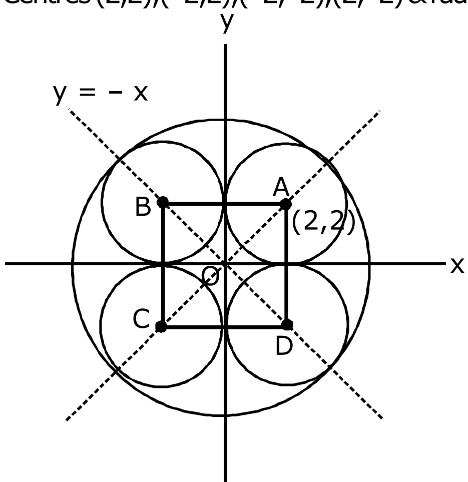
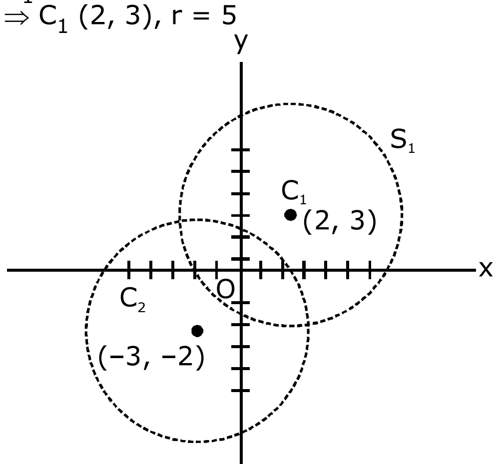
EXERCISE – II HINTS & SOLUTIONS

Sol.1 Centres (2,2),(–2,2),(–2,–2),(2,–2) & radius =2



(A) Centres lies on $y^2 - x^2 = 0$
(B) not only y = x
(C) Area of quadrilateral ABCD
$= 4 \times 4 = 16$ sq. units.
(D) Radius of such circle = OA + 2
$= \sqrt{2^2 + 2^2} + 2 = 2\sqrt{2} + 2$
$= 2(\sqrt{2} + 1)$
Area $= \pi 2^2 (\sqrt{2} + 1)^2 = \pi 4(3 + 2\sqrt{2})$

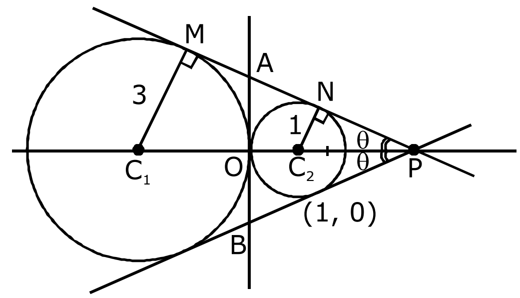
Sol.2 $S_1 \equiv x^2 + y^2 - 4x - 6y - 12 = 0$
$\Rightarrow C_1 (2, 3), r = 5$



$S_2 \equiv x^2 + y^2 + 6x + 4y - 12 = 0$
$C_2 (-3, -2), r = 5$
$L = x + y = 0$
$S_1 - S_2 = 0$
$-10x - 10y = 0$
$\Rightarrow x + y = 0$
(A) Origin inside both cirlce
(B) L is common chord
(C) L is radical Axis

(D) $m_{C_1C_2} = \frac{5}{5} = 1$ & $m_L = -1$
$C_1C_2 \perp L$

Sol.3 $S_1 \equiv x^2 + y^2 + 6x = 0$
$\Rightarrow C_1(-3, 0), r_1 = 3$
$S_2 \equiv x^2 + y^2 - 2x = 0$
$\Rightarrow C_2 (1, 0), r_2 = 1$
$C_1C_2 = 4$ $\quad r_1 + r_2 = 4$
$C_1C_2 = r_1 + r_2$ (A)
S_1 & S_2 touch each other externally



$$\frac{PC_1}{PC_2} = \frac{3}{1}$$

$$PO\left(\frac{(-3)1 - (1)3}{1-3}, 0\right) \equiv P(3, 0)$$

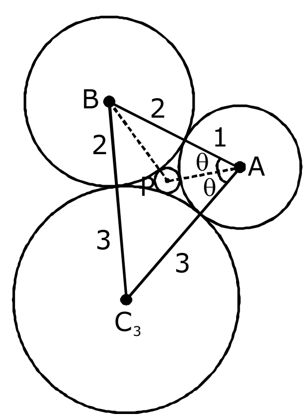
$OP = 3, OC_2 = 1, C_2P = 2$

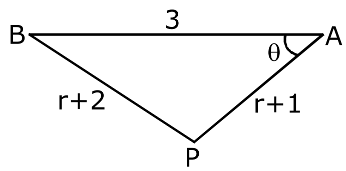
In $\Delta C_2NP \Rightarrow \frac{1}{2} = \sin\theta \Rightarrow \theta = 30^\circ$

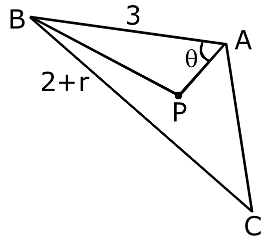
$$\frac{OA}{OP} = \tan 30^\circ \Rightarrow OA = \frac{3}{\sqrt{3}} \Rightarrow OA = \sqrt{3}$$

Area of $\Delta PAB = \frac{1}{2} AB \times OP = \frac{1}{2} \times 2\sqrt{3} \times 3 = 3\sqrt{3}$ (C)

Sol.4 (4) a = 5, b = 4, c = 3
which is right angled Δ
at A
$\angle PAB = \theta, \angle PAC = \alpha$
$\theta + \alpha = 90^\circ$
In ΔABP







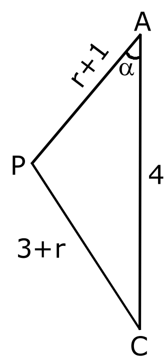
$$\cos\theta = \frac{9+(r+1)^2-(r+2)^2}{2.3.(r+1)}$$

$$= \frac{9+r^2+2r+1-r^2-4r-4}{6(r+1)} = \frac{6-2r}{6(r+1)}$$

$$\Rightarrow \cos\theta = \frac{3-r}{3(1+r)} \quad (A)$$

In ΔACP

$$\cos\alpha = \frac{16+(r+1)^2-(3+r)^2}{2.4.(r+1)}$$



$$= \frac{16+r^2+2r-1-9-6r-r^2}{2.4(r+1)}$$

$$= \frac{8-4r}{8(r+1)} = \frac{(2-r)}{2(1+r)}$$

$\theta + \alpha = 90^o$

$\theta = 90 - a \Rightarrow \cos\theta = \sin\alpha$

$$\Rightarrow \cos^2\theta = \sin^2\alpha = \frac{(3-r)^2}{9(1+r)^2} = \frac{4(1+r)^2-(2-r)^2}{4(1+r)^2}$$

$\Rightarrow \quad 4(9 - 6r + r^2) = 9 [4 + 8r + 4r^2 + 4r - r^2]$

$\Rightarrow \quad 36 - 24r + 4r^2 = 108r + 27r^2$

$\Rightarrow \quad 23r^2 + 132 r - 36 = 0$

$\Rightarrow \quad (r + 6) (23r - 6) = 0$

$\Rightarrow \quad r = \dfrac{6}{23}$

$\because \quad r + 6 \neq 0$

Sol.5 $(x - r)^2 + y^2 = r^2$

$\Rightarrow \quad x^2 + y^2 - 2xr = 0$

8 tangent at (x_1, y_1)

$xx_1 + yy_1 - r (x + x_1) = 0$

$(x_1 - r) x + yy_1 - rx_1 = 0$

$$\text{slope } m_T = \frac{r-x_1}{y_1} = \frac{r-x}{y} \quad (B)$$

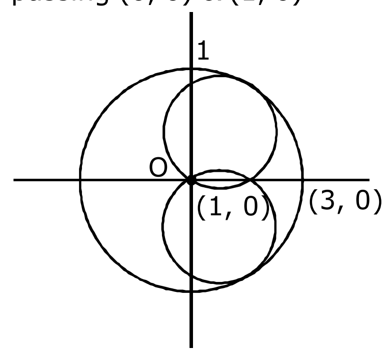
$$\frac{r-x}{y} = \frac{2xr-2x^2}{2xy}$$

$$= \frac{x^2+y^2-2x^2}{2xy} = \frac{y^2-x^2}{2xy} \quad (C)$$

Sol.6 Let circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

passing (0, 0) & (1, 0)



$C = 0 \quad 1 + 2q = 0 \quad \Rightarrow \quad g = -\dfrac{1}{2}$

Circle will be

$x^2 + y^2 - x + 2fy = 0$

$$\left(\frac{1}{2}, -f\right), \; r_1 = \sqrt{f^2+\frac{1}{4}}$$

touches internally

$x^2 + y^2 = 9$, (0, 0), $r_2 = 3$

$$\sqrt{\left(\frac{1}{2}\right)^2 + f^2} = \left|3-\sqrt{f^2+\frac{1}{4}}\right| \quad \left\{\because \; 3 > \sqrt{f^2+\frac{1}{4}}\right.$$

$$\frac{1}{4} + f^2 = \left(3-\sqrt{f^2+\frac{1}{4}}\right)^2$$

$$\Rightarrow \frac{1}{4} + f^2 = 9 + f^2 + \frac{1}{4} - 6\sqrt{f^2+\frac{1}{4}}$$

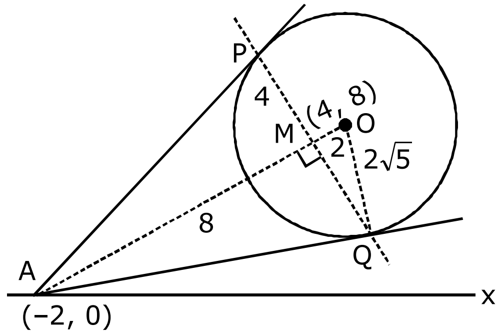
$$\Rightarrow \sqrt{f^2+\frac{1}{4}} = \frac{3}{2} \Rightarrow f^2 + \frac{1}{4} = \frac{9}{4}$$

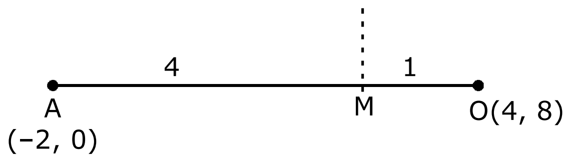
$\Rightarrow f^2 = 2 \quad \Rightarrow f = \pm\sqrt{2}$

Centres are $\left(\dfrac{1}{2}, \pm\sqrt{2}\right)$

Sol.7 $(x-4)^2 + (y-8)^2 = 20$

$x^2 + y^2 - 8x - 16y + 60 = 0$





C.O.C.

$-2x - 4(x-2)\ (x-2) - 8\ (y+0) + 60 = 0$

$-6x - 8y + 68 = 0$

$\Rightarrow \quad 3x + 4y - 34 = 0$

$AO = \sqrt{6^2 + 8^2} = 10$

$OM = \dfrac{12x + 32 - 34}{\sqrt{3^2 + 4^2}} = \dfrac{10}{5} = 2$

$M\left(\dfrac{14}{5}, \dfrac{32}{5}\right)$

$PM = \sqrt{20 - 4} = \sqrt{16} = 4$

$C.O.C = \tan\theta = \dfrac{-3}{4}$

$\Rightarrow \quad \sin\theta = \dfrac{3}{5}, \cos\theta = \dfrac{-4}{5}$

in parametric form

$$\frac{x - \frac{14}{5}}{-\frac{4}{5}} = \frac{y - \frac{32}{5}}{\frac{3}{5}} = \pm 4$$

$\Rightarrow \quad \dfrac{5x - 14}{-4} = \dfrac{5y - 32}{3} = \pm 4$

$\Rightarrow \quad 5x = 14 - 16, 5y = 32 + 12$

$x = -\dfrac{2}{5}, y = \dfrac{44}{5}$

$\left(\dfrac{-2}{5}, \dfrac{44}{5}\right)$

$5x = 14 + 16$, $5y = 32 - 12$

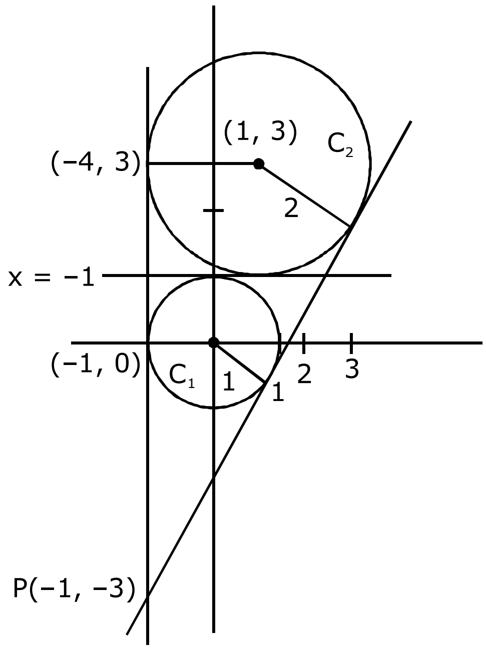
$x = 6, \qquad y = 4$

$(6, 4)$

Sol.8 $x^2 + y^2 = 1, \quad C_1(0, 0), r_1 = 1$

$x^2 + y^2 - 2x - 6y + 6 = 0, C_2\ (1, 3), r_2 = 2$

$\dfrac{C_1P}{C_2P} = \dfrac{1}{2}$



O is mid point of PC_2

P(–1, –3)

D.C.T.

$y + 3 = m(x+1) \Rightarrow mx - y + m - 3 = 0$

$1 = \dfrac{|m-3|}{\sqrt{m^2+1}}$

$\Rightarrow \quad m^2 + 1 = m^2 + 1 = m^2 - 6m + 9$

$m = \dfrac{4}{3}$ & $m = \infty$

$x = -1$ & $4x - 3y - 5$

$Q.\ \left(\dfrac{1.1 + 2.0}{3}, \dfrac{3.1 + 2.0}{3}\right) \equiv \left(\dfrac{1}{3}, 1\right)$

T.C.T.

$y - 1 = m\left(x - \dfrac{1}{3}\right)$

$\Rightarrow \quad 3mx - 3y + 3 - m = 0$

$1 = \dfrac{|3-m|}{\sqrt{9m^2 + 9}}$

$\Rightarrow \quad 9m^2 + 9 = m^2 - 6m + 9$

$\Rightarrow \quad 8m^2 + 6m = 0$

$m = 0, m = -\dfrac{3}{4}$

$y = 1$ & $3x + 4y - 5 = 0$